Limiting Performance of Ground Transportation Vehicles Subject to Transient Loading

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Theme

A COMPUTATIONAL approach for the study of potential physical performance of ground transportation vehicles subject to transient disturbances is demonstrated. In particular, the problem of limiting performance of vehicles in protecting passengers or freight under crash conditions or from rough terrain is considered. While present results are based on Ref. 1, related work can be found in Refs. 2–4.

Contents

For numerous instances in the study of the dynamics of ground transportation systems it is valuable to know the true optimal performance. That is, the limiting performance is desired regardless of the design configuration of those portions of the system under consideration, e.g., suspension systems. A computational approach for determining this limiting performance of vehicles subject to transient disturbances based on response variable criteria is set forth in this Synoptic.

For the purpose of a limiting performance study, the transportation system dynamics are described using the second- or first-order equations

$$\mathbf{M}\ddot{\bar{q}}+\mathbf{C}\dot{\bar{q}}+\mathbf{K}\bar{q}+\mathbf{U}\bar{u} = \mathbf{F}\bar{f}_k \tag{1}$$

$$\dot{\bar{s}} = \mathbf{A}\bar{s}+\mathbf{B}\bar{u}+\mathbf{D}\bar{f}_k \tag{2}$$

in which $\bar{u}$ is a vector of time-varying functions (forces or moments), called control or isolator forces, that have replaced portions of the physical system; **M, C, K, U, F, A, B, D** are coefficient matrices; $\bar{q}$ and $\bar{s}$ are vectors of response variables, e.g., displacements, stresses; and $\bar{f}_k$ is a forcing function vector where the subscript indicates the kth set of forcing or loading functions.

These equations of motion appear to be linear. However, in fact, they are "quasi-linear" since those portions of the system replaced by $\bar{u}$ can be linear, nonlinear, active, or passive. The remainder of the system must be linear, as must the over-all kinematics.

The limiting performance problem is to find the $\bar{u}$ of Eqs. (1) or (2) such that bounds imposed on some of the response variables $\bar{s}$ or $\bar{q}$ or control forces $\bar{u}$ are not violated while the maximum (or minimum) in time of other elements of $\bar{s}$ or $\bar{q}$ are minimized (or maximized).

A computer system, PERFORM, has been developed[3,4] to solve the limiting performance problem for dynamic systems. PERFORM is available from COSMIC. For prescribed initial conditions the $\bar{u}$ vector is computed by PERFORM such that the maximum in time ϕ of

$$\bar{\psi} = |\mathbf{G}\bar{s}\ (\text{or}\ \bar{q})+\mathbf{H}\bar{u}+\mathbf{I}\bar{f}_k| \tag{3}$$

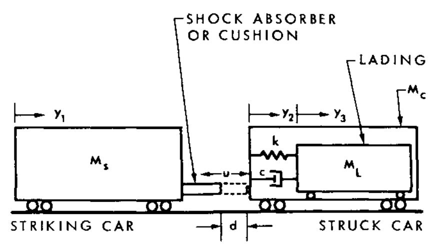
is minimized. **G**, **H**, and **I** are coefficient matrices. Also, equality or inequality constraints placed on a relationship of the form of Eq. (3) can be imposed.

The computations are performed as a linear programing problem. The details of this formulation are given in Refs. 2 and 4. Briefly, the computation is performed by discretizing all variables in time, e.g., the calculations here employed a piecewise constant $\bar{u}$, and integrating Eqs. (1) or (2). Then, using linear programing, the discrete elements of $\bar{u}$ are found such that ϕ is minimized subject to response constraints and the performance constraint $\bar{\psi}_i \leqq \phi$, $i = 1, 2, \ldots, N$, where i indicates discrete instants of time. All rows of Eq. (3) are treated simultaneously.

In contrast to most optimization schemes which require multiple analyses of the system dynamics, a single solution of Eqs. (1) or (2) suffices to establish the linear programing problem for a limiting performance study. The dimension of the linear programing problem depends on the number of isolators, the number of time intervals, and the number of sets of forcing functions. It is independent of the degrees of freedom of the system. Since standard linear programing software capabilities are of such high capacity, large problems can be solved efficiently.

As an example of the limiting performance of vehicles in protecting passengers or cargo under crash conditions, we consider the problem of lading damage of a rail vehicle that is struck by another vehicle.

Typical analyses of the freight car lading protecting problem are described in Refs. 5 and 6. The dynamic system model of Fig. 1 is taken from Ref. 4. The car containing the lading is resting freely on a horizontal track. Here, u is a function of time which is in contrast to Ref. 5 where u was taken as a constant. The generic or control force u can represent any cushioning configuration. The equations of motion for this system are



c = DAMPING COEFFICIENT, DUE TO FRICTION IN LADING
k = ELASTIC CONSTANT, REPRESENTING RESILIENCE IN LADING
M_c = MASS OF STRUCK CAR BODY
M_L = MASS OF LADING
M_s = MASS OF STRIKING CAR
u = CUSHION FORCE
d = $y_1 - y_2$ = CUSHION TRAVEL

Fig. 1 Model of striking car, struck car, and lading.

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Index categories: Structural Dynamic Analysis; Navigation, Control, and Guidance Theory.

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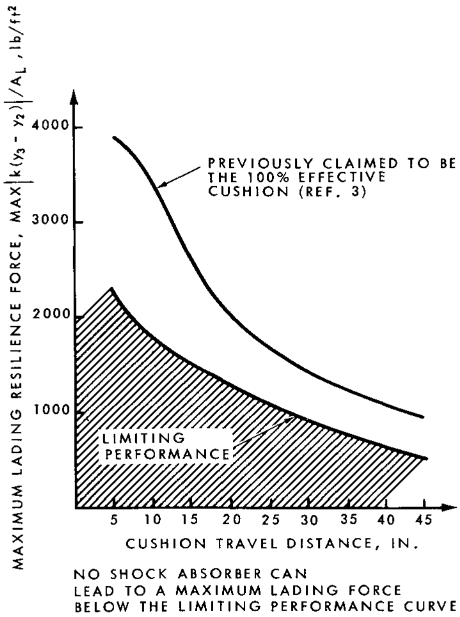

Fig. 2 Limiting performance of freight car struck by a moving car with $V = 10$ mph.

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\begin{aligned}
M_s \ddot{y}_1 + u &= 0 \\
M_c \ddot{y}_2 - k(y_3 - y_2) - c(\dot{y}_3 - \dot{y}_2) - u &= 0 \qquad (4) \\
M_L \ddot{y}_3 + k(y_3 - y_2) + c(\dot{y}_3 - \dot{y}_2) &= 0
\end{aligned}
$$

where y_1, y_2, y_3 are displacements measured from the impact point of the striking car, struck car, and lading, respectively.

For specified impact conditions, the problem is to minimize the peak resilience force transmitted to the lading while the cushion travel distance is bounded. Thus, the u is sought that minimizes

$$\max |k(y_3 - y_2)| \qquad (5)$$

with the restriction that

$$0 \leqq d \leqq A \qquad (6)$$

where A is prescribed.

A tradeoff between the maximum cushion travel distance and the minimal peak force transmitted to the lading is shown in Fig. 2 for a moving car velocity of 10 mph. In this linear programing computation, the following data were used: $c = 23{,}000$ lb sec/ft, $k = 342{,}000$ lb/ft, $M_c = 1630$ lb sec^2/ft, $M_L = 5280$ lb sec^2/ft, A_L (lading contact area) = 33.7 ft^2.

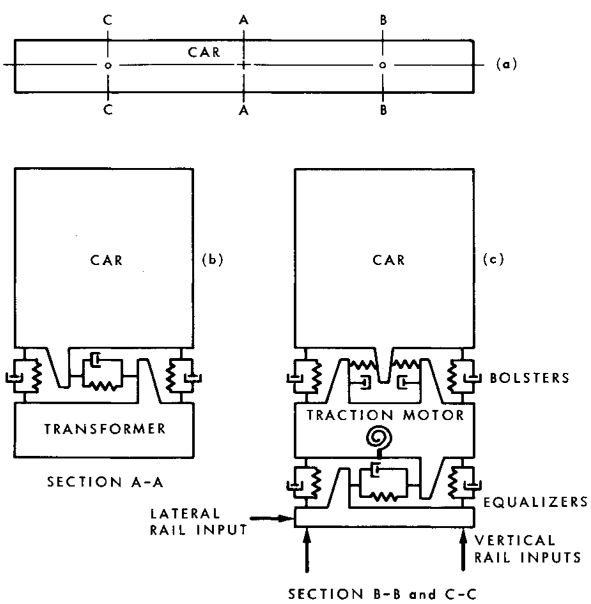

Fig. 3 Mathematical lateral model of railroad car and truck suspension: a) top view; b) midsection view; c) truck suspension (Sec. B-B and C-C).

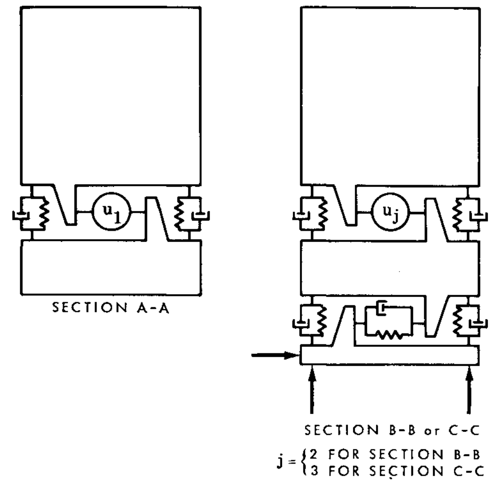

Fig. 4 Rail car model with inclusion of three isolator forces.

Regardless of the design, it is not possible to construct a cushion that can reduce the transmitted force below the limiting performance curve for the given initial velocity. Also shown in Fig. 2 is the performance curve taken from Ref. 5 for a constant cushion force. Heretofore it has been contended (Refs. 5 and 6) that this represents the performance of a 100% efficient cushion. However, as is seen in Fig. 2, substantial improvement in performance can be achieved. Furthermore, in contrast to previous contentions that the optimum cushion would be a constant absorber force, the absolute optimum force is in fact of the piecewise-constant type.

Many other limiting performance problems can be formulated. Reference 1 includes results from a system that can sense its environment and prepare for its own protection, that is, an early-warning or preview-absorber system.

As another example, consider the ten-degree-of-freedom model of a high-speed rail car (Fig. 3), which was employed in Ref. 7 for finding the lateral responses due to lateral or rolling inputs from the rail. For a limiting performance study, the spring-dashpot suspension systems are replaced by control or isolator forces. This is shown for the case of three isolators in Fig. 4. With the system equations of motion of the configuration of Fig. 4 placed in the form of Eq. (1), the problem of finding the characteristics of the perfect suspension system such that a maximum lateral acceleration is minimized while lateral deflections of the bolster springs are bounded, can be solved.

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